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Magma flow between summit and Pu‘u ‘Ō‘ō at Kīlauea Volcano, Hawai‘i

C. P. Montagna
Department of Earth Science, Keith-Wiess Geological Lab, Rice University, Houston, Texas, USA (montagna@pi.ingv.it)

Now at Istituto Nazionale di Geofisica e Vulcanologia, Sezione di Pisa, Pisa, Italy

H. M. Gonnermann
Department of Earth Science, Keith-Wiess Geological Lab, Rice University, Houston, Texas, USA

[1] Volcanic eruptions are often accompanied by spatiotemporal migration of ground deformation, a consequence of pressure changes within magma reservoirs and pathways. We modeled the propagation of pressure variations through the east rift zone (ERZ) of Kīlauea Volcano, Hawai‘i, caused by magma withdrawal during the early eruptive episodes (1983–1985) of the ongoing Pu‘u ‘Ō‘ō-Kupaianaha eruption. Eruptive activity at the Pu‘u ‘Ō‘ō vent was typically accompanied by abrupt deflation that lasted for several hours and was followed by a sudden onset of gradual inflation once the eruptive episode had ended. Similar patterns of deflation and inflation were recorded at Kīlauea’s summit, approximately 15 km to the northwest, albeit with time delays of hours. These delay times can be reproduced by modeling the spatiotemporal changes in magma pressure and flow rate within an elastic-walled dike that traverses Kīlauea’s ERZ. Key parameters that affect the behavior of the magma-dike system are the dike dimensions, the elasticity of the wall rock, the magma viscosity, and to a lesser degree the magnitude and duration of the pressure variations themselves. Combinations of these parameters define a transport efficiency and a pressure diffusivity, which vary somewhat from episode to episode, resulting in variations in delay times. The observed variations in transport efficiency are most easily explained by small, localized changes to the geometry of the magma pathway.

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1. Introduction

[2] Magma movements in reservoirs and pathways beneath active volcanoes cause changes in the pressure of those systems. On eruptive time scales, the surrounding rock deforms elastically in response to such pressure variations, which are thus transmitted to the Earth’s surface, where they can be recorded as ground deformation and seismic signals [*Dzurisin*, 2000, 2003; *Segall*, 2010].

[3] The interaction of the ambient stress field with dike propagation has been extensively studied, and much effort has been directed toward understanding dike ascent and propagation mechanisms [e.g., *Weertman*, 1971; *Spence and Turcotte*, 1985; *Rubin*, 1993; *McLeod and Tait*, 1999; *Segall et al.*, 2001; *Jellinek and DePaolo*, 2003; *Rivalta and Segall*, 2008; *Karlstrom et al.*, 2009]. Models of dike propagation, which couple the flow dynamics to the elastic properties of the crust, indicate that the surrounding wall rock exerts a normal stress on the dike. This stress is proportional to the dike opening, provided that the dike itself is elongated in one direction compared to the other two [*Lister and Kerr*, 1991; *Pinel and Jaupart*, 2000; *Bokhove et al.*, 2005; *Woods et al.*, 2006]. Attention has also been directed toward the study of subhorizontal, long-lived dikes [*Wilson and Head*, 1988; *Mériaux and Jaupart*, 1995; *Pinel and Jaupart*, 2004; *Costa et al.*, 2007], which may be part of the magmatic systems of basaltic volcanoes with long rift zones such as Kīlauea [*Parfitt and Wilson*, 1994; *Segall et al.*, 2001; *Montgomery-Brown et al.*, 2010], Krafla [*Einarsson and Brandsdottir*, 1980; *Tryggvason*, 1986; *Buck et al.*, 2006], or Afar [*Grandin et al.*, 2012].

[4] Building upon the work of *Pinel and Jaupart* [2000] and *Bokhove et al.* [2005], we study how variations in magma pressure and flow rate, caused by changes in magma withdrawal or supply, are transmitted in space and time through a horizontal dike surrounded by linearly elastic wall rock. We evaluate how the propagation rate and attenuation of such changes in pressure and flow rate depend on the physical characteristics of the system (rock elasticity, magma viscosity, dike dimensions), as well as on the pressure changes themselves (magnitude, duration).

[5] We apply this elastic dike model to the early episodes (1983–1985) of the Pu‘u ‘Ō‘ō-Kupaianaha eruption at Kīlauea Volcano, Hawai‘i. Although the model is a geometrically idealized representation of the complex magmatic pathway that traverses Kīlauea's east rift zone (ERZ), it reproduces the fundamental patterns of deflation and inflation at Pu‘u ‘Ō‘ō and at Kīlauea's summit. Tilt measurements show that the onset of eruptive episodes at the Pu‘u ‘Ō‘ō vent [*Heliker et al.*, 2003] was typically accompanied by abrupt deflation and followed by sudden onset of gradual inflation during repose periods [*Wolfe et al.*, 1987]. Approximately 15 km to the northwest, Kīlauea's summit underwent similar patterns of deflation and inflation, albeit with a time delay of several hours [*Dvorak and Okamura*, 1985; *Wolfe et al.*, 1987].

[6] In section 2, we introduce the governing equation for the dike model and perform a parametric analysis, wherein we illustrate the parameter dependence of the rate at which pressure perturbations travel along the length of the dike. In section 3.1, selected episodes from the early stage of the Pu‘u ‘Ō‘ō-Kupaianaha eruption are presented, and in section 3.2, delay times are introduced. In sections 3.3 and 3.4, the model is applied to episode 18 of the Pu‘u ‘Ō‘ō-Kupaianaha eruption, and in section 3.5, we show how model parameters affect the observed variability in delay times.

2. Elastic Dike Model

2.1. Governing Equations

[7] We investigate horizontal magma flow through a dike of variable width, b. The dike is assumed to have an elliptical cross section and b is much smaller than the constant horizontal length H, as well as the vertically oriented major axis, a. The problem can be treated as one-dimensional in the horizontal direction x. Consequently, all quantities vary only along x, including the width-averaged horizontal velocity of magma flow, u [*Pinel and Jaupart*, 2000; *Bokhove et al.*, 2005; *Woods et al.*, 2006].

[8] In addition, we assume that the magma viscosity, μ, is constant and Newtonian, and that the flow is isothermal [*Pinel and Jaupart*, 2000; *Bokhove et al.*, 2005; *Woods et al.*, 2006; *Costa et al.*, 2007]. We also neglect magma compressibility (Appendix A) and thus assume a constant magma density, ρ_m.

[9] The mass and momentum conservation equations that describe this idealized system are [*Bokhove et al.*, 2005]

$$\partial_t(\rho_m b) + \partial_x(\rho_m b u) = 0, \tag{1}$$

$$\rho_m \partial_t u + \rho_m u \partial_x u = -\partial_x p - \frac{\gamma \mu u}{b^2}, \tag{2}$$

where p is magma pressure (see Table 1 for definitions of variables and parameters). The last term of equation (2) represents viscous friction, evaluated for an incompressible, steady flow. The coefficient $\gamma = 12$ is appropriate in the approximation of flow between two parallel plates, and its derivation is provided in Appendix B. For smaller aspect ratios, a/b, other values of γ apply [*Pinel and Jaupart*, 2000].

[11] The system of equations (1) and (2) is closed by relating the dike width to the magma pressure at the dike walls. For an elliptical dike cross section and a linearly elastic surrounding material [*Muskhelishvili*, 1963; *Pinel and Jaupart*, 2000]

$$p = \frac{2Gb}{2a(1-\nu)+b(1-2\nu)}, \tag{3}$$

where ν is the Poisson's ratio for the host rock and G is the shear modulus. If the aspect ratio a/b is large, this relation reduces to the dike width b being proportional to p, that is

$$b = \lambda p. \tag{4}$$

Table 1. Definition of Symbols (hat indicates a dimensionless quantity).

Name	Description
a	dike height
B_0	characteristic dike width
b	dike width
b_0	initial dike width
G	shear modulus of host rock
H	dike length
L_0	characteristic dike length
P	perturbation amplitude
p	fluid pressure
p_0	initial fluid pressure
p_H	pressure in the Halema'uma'u reservoir
Q_e	eruptive flow rate
Q_{Puu}	magma flow rate from the dike into the Pu'u 'Ō'ō reservoir
q	magma flow rate in the dike
T_0	characteristic time scale
T_E	eruption duration
T_P	perturbation duration
t	time
u	dike width averaged flow velocity
V	volume
V_H	volume of the Halema'uma'u reservoir
x	horizontal coordinate
β	equation parameter
β_d	compressibility of the elastic dike
β_m	magma compressibility
γ	friction coefficient
Δb	dike width variation
Δp	amplitude of magma pressure variation
ΔV	volume variation
$\Delta\tau$	delay time
$\Delta\tau_d$	delay time for the onset of deflation
$\Delta\tau_i$	delay time for the onset of reinflation
$\Delta\psi$	amplitude of tilt variation
κ	pressure diffusivity
$\Lambda\square$	compressibility ratio
λ	elasticity of the host rock
μ	magma viscosity
ν	Poisson's ratio for host rock
ρ_m	magma density
τ_d	start of fountaining episode
τ_i	end of fountaining episode
ϕ	transport efficiency
ψ	tilt
ψ_{smooth}	smoothed tilt
ω	wavelength of pressure pertubation

[12] Here, $\lambda = a(1-\nu)/G$ represents the elasticity of the host medium. Equation (3) holds strictly only for pressure variations with wavelengths that are greater than the smaller between dike height and length [*Dunham and Ogden*, 2012], and far from the tip of the dike. Otherwise, the coupling between p and b becomes nonlinear and nonlocal [*Lister*, 1990]. Local coupling as assumed here represents a first-order approximation, valid only for long-wavelength perturbations.

[13] The magma flow rate through Kīlauea's ERZ is small enough for the inertial term in equation (2) to be negligible (Appendix C). The velocity is thus given by [*Bokhove et al.*, 2005]

$$u = -\frac{b^2}{\gamma\mu}\partial_x p. \tag{5}$$

[14] Upon substitution into equation (1), this yields

$$\partial_t p - \partial_x\left(\beta b^3 \partial_x p\right) = 0, \tag{6}$$

where

$$\beta = \frac{1}{\gamma\mu\lambda}. \tag{7}$$

[15] Equation (6) is a nonlinear diffusion equation that governs the spatiotemporal change in pressure within the elastic-walled dike.

2.2. Pressure Diffusivity and Transport Efficiency

[16] The key parameter that determines the rate of propagation for pressure changes, as they travel along the dike, is pressure diffusivity, which can be defined as

$$\kappa(p) = \beta b^3. \tag{8}$$

[17] A second parameter of importance for magma transport is the proportionality constant between magma flow rate and pressure gradient. It represents a transport efficiency and can be defined as

$$\phi(p) = \beta b^2. \tag{9}$$

[18] Both κ and ϕ are strongly dependent on variations of the dike width b.

2.3. Dimensionless Form

[19] Equation (6) can be nondimensionalized by means of a characteristic dike width, B_0, dike length, L_0, and magma travel time, T_0, resulting in

$$\partial_{\hat{t}}\hat{p} - \partial_{\hat{x}}\left(\hat{b}^3\partial_{\hat{x}}\hat{p}\right) = 0. \tag{10}$$

[20] Here, $\hat{p} = p/\lambda B_0, \hat{b} = b/B_0, \hat{x} = x/L_0$, and $\hat{t} = t/T_0 = t\beta B_0^3/L_0^2$.

[21] To evaluate how a well established and persistently open dike responds to relatively small variations of magma pressure, we rewrite equation (4) by separating the initial dike width b_0 and pressure p_0 from their variations Δb and Δp, resulting in

$$b = b_0 + \Delta b = b_0 + \lambda\Delta p. \tag{11}$$

[22] The importance of the dike width, b_0, prior to an eruptive pressure change can thus be separated from the effects caused by local variations in dike width caused by the propagation of pressure changes, Δp. We will solve equation (10) for different combinations of b_0 and Δp in order to highlight their different contributions.

2.4. General Solutions and Parameter Dependencies

2.4.1. Boundary Conditions

[23] To examine the response of the dike model to the aforementioned changes in pressure, we use an idealized model configuration. We impose a sinusoidal pressure change of amplitude P and period $2T_P$ as a time-dependent boundary condition at $x = 0$

$$p(t) = \begin{cases} p_0 - P\sin(\pi t/T_P) & 0 \le t \le T_P \\ p_0 & t > T_P. \end{cases} \tag{12}$$

[24] At the opposite end of the dike, $x = L_0$, we use a Neumann boundary condition

$$\partial_x p = 0 \quad t \ge 0, \tag{13}$$

which allows p to vary with time, in response to the propagation of the imposed pressure change within the dike. This model configuration isolates the response of the dike to an imposed pressure change at one end.

2.4.2. Delay Time

[25] A key observation is the time delay in deflationary or inflationary tilt changes between Puʻu ʻŌʻō and Kīlauea's summit. For the purpose of this general analysis, we define the delay time as the time it takes for the minimum of the pressure variation to reach the other side of the dike. In the subsequent sections where we apply this model to tilt data from individual eruptive episodes, the definition of delay time will be somewhat different.

2.4.3. Parameter Dependencies

[26] Equation (10) is solved numerically using the pdepe partial differential equation solver of MATLAB®. The solver used is ode15s, which is an implicit solver, suitable for stiff problems where the time step and the integration formula adapt dynamically, to deal with rapid changes in the solution [*Shampine and Reichelt*, 1997]. The accuracy in space is second order [*Skeel and Berzins*, 1990].

[27] Two illustrative examples for the spatiotemporal propagation of a pressure perturbation are shown in Figure 1. All else being the same, the initial sinusoidal pressure change becomes more attenuated in amplitude and propagates at a lower speed with decreasing initial dike width $\hat{b}_0$. This is a consequence of both κ and ϕ (equations (8) and (9)) being smaller.

[28] We show in Figure 2 how the dimensionless delay time $\Delta\hat{\tau} = \Delta\tau/T_0$ changes with respect to $\hat{b}_0 = b_0/B_0, \Delta\hat{p} = \Delta p/\lambda B_0$, and the dimensionless duration of the pressure perturbation, $\hat{T}_P = T_P/T_0$. Because κ scales as b^3 and ϕ as b^2, the value of $\hat{b}_0$ most strongly affects the rate at which the pressure variation travels through the dike, with a decrease (increase) in $\hat{b}_0$ resulting in longer (shorter) delay times. $\Delta\hat{\tau}$ is less affected by the amplitude of the perturbation, but does increase with increasing $\Delta\hat{p}$. For values of $\hat{T}_P < 0.5$, the delay time also increases with decreasing $\hat{T}_P$.

3. Application of the Elastic Dike Model to Kīlauea's ERZ

3.1. Early Episodes of the Puʻu ʻŌʻō-Kupaianaha Eruption

[29] Although the elastic dike model is an idealization of a more complex east-rift zone plumbing system, we apply it to the dike through which magma is thought to have flowed from Kīlauea's summit to the Puʻu ʻŌʻō eruptive vent during the early high fountaining episodes (1983–1985) of the Puʻu ʻŌʻō-Kupaianaha eruption (Figure 3) [*Dvorak and Okamura*, 1987; *Wolfe et al.*, 1987; *Okamura et al.*, 1988; *Parfitt and Wilson*, 1994; *Heliker et al.*, 2003].

[30] Before 1983, magma had been accumulating in the ERZ with successive dike intrusions since

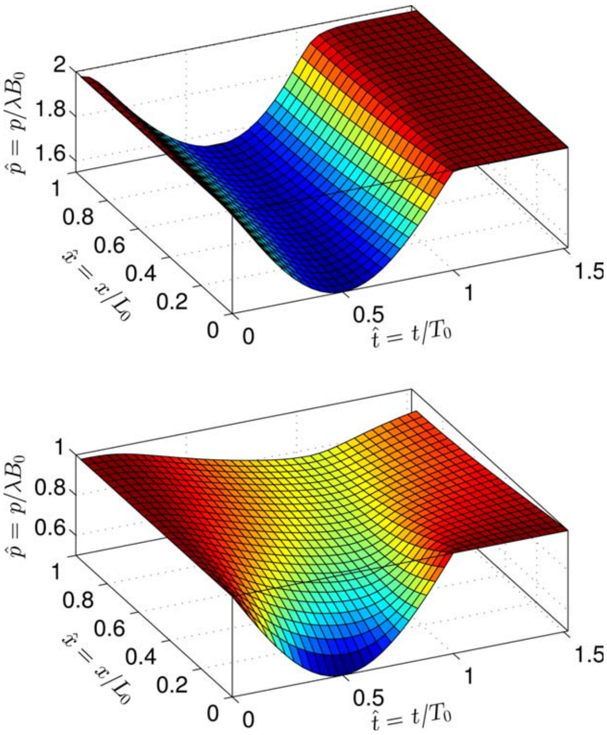

Figure 1. Propagation of an idealized sinusoidal change in pressure using equation (10). Boundary condition at $\hat{x} = 0$ is $\hat{p}(\hat{t}) = \hat{p}_0 - \Delta\hat{p}\sin(\pi\hat{t}/\hat{T}_P)$. Here, $\hat{p} = p/\lambda B_0$ is dimensionless pressure, $\hat{t} = t/T_0$ dimensionless time, and $\hat{x} = x/L_0$ dimensionless distance along the length of the dike. In both cases, the dimensionless duration of the pressure change at the boundary $\hat{x} = 0$ has a value of 1 and dimensionless magnitude of pressure change $\Delta\hat{p} = 0.5$. (a) The dimensionless unperturbed dike width is $\hat{b}_0 = 2$ and (b) $\hat{b}_0 = 1$. Note the

1956 [*Jackson*, 1988; *Wilson and Head*, 1988; *Cayol et al.*, 2000]. The intrusion on 3 January 1983 [*Okamura et al.*, 1988; *Heliker and Mattox*, 2003] initiated the ongoing ERZ eruption. Dimensional estimates of the dike, which hydraulically connected Kīlauea's summit with a shallow magma body beneath the Pu'u 'Ō'ō vent [*Dvorak and Okamura*, 1987], fall in the range 2–3.5 m for width and 2.4–4.4 km for hight [*Wolfe et al.*, 1987; *Okamura et al.*, 1988; *Parfitt and Wilson*, 1994; *Heliker et al.*, 2003]. Seismicity indicated that this dike had propagated more or less horizontally [*Wolfe et al.*, 1987], thus justifying the idealized geometry of a horizontal dike for our model.

[31] The model objective is to constrain the characteristic transport properties of the ERZ magmatic system by using equation (6) to model the observed changes in tilt, at both Pu'u 'Ō'ō and Kīlauea's summit, in response to individual eruptive episodes. In particular, we focus on the difference in time delays between the onset of deflation at Pu'u 'Ō'ō and Kīlauea's summit as well as the subsequent onset of reinflation.

[32] The eruptive episodes and associated deflation at Pu'u 'Ō'ō typically lasted less than 24 h [*Heliker and Mattox*, 2003]. They were characterized by average eruption rates of about 10^6 m^3 h^{-1} and by lava fountaining with heights of up to 500 m. During repose periods, averaging approximately 24 days, tilt measurements at Pu'u 'Ō'ō recorded a gradual reinflation beginning at the end of the eruption.

[33] Deflation and inflation at Pu'u 'Ō'ō were a consequence of magma withdrawal and accumulation, respectively, within a magma body that may

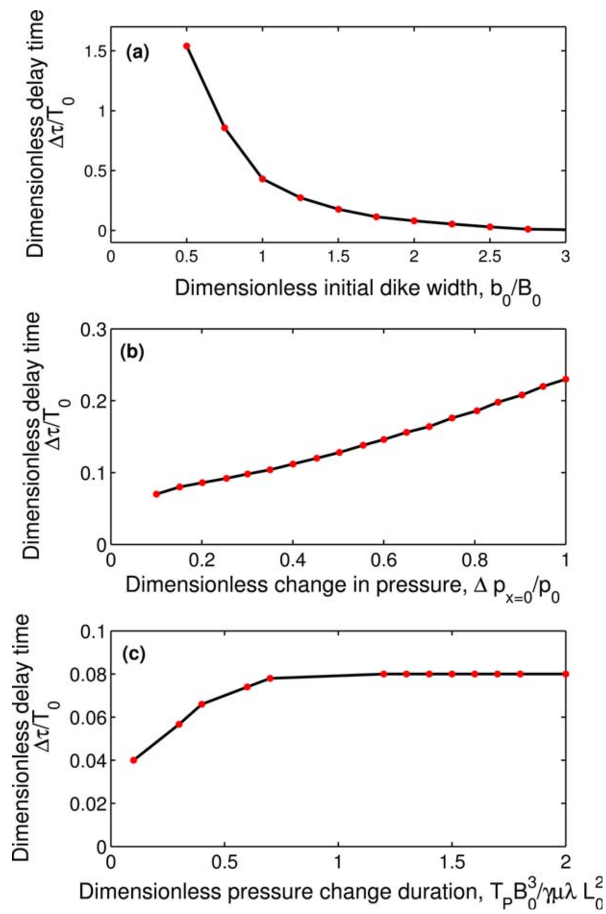

Figure 2. Parameter dependence of dimensionless delay time, $\Delta\tau/T_0$, the time for the minimum of the pressure variation to reach the other side of the dike. (a) Delay time versus dimensionless initial dike width, b_0/B_0. (b) Delay time versus dimensionless change in pressure $\Delta p_{x=0}/(\lambda B_0)$, where $x = 0$ denotes the end of the dike where the perturbation is imposed

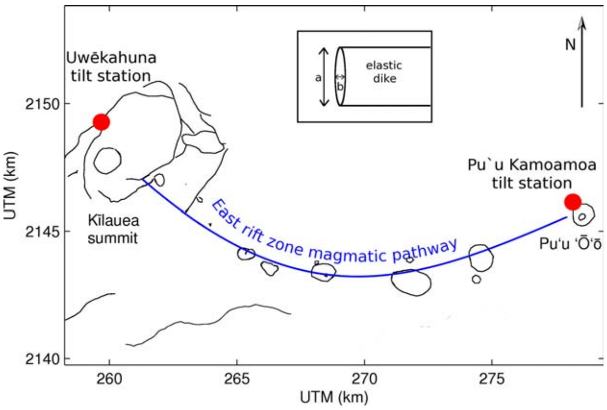

Figure 3. Schematic diagram of the ERZ magma pathway connecting Kīlauea's summit and the Pu'u 'Ō'ō vent.

have been a remnant of the first intruded dike [*Wilson and Head*, 1988; *Hoffmann et al.*, 1990; *Garcia et al.*, 1992; *Mittelstaedt and Garcia*, 2007]. The precise geometry of this magma body remains uncertain in detail [*Heliker et al.*, 2003].

[34] The pattern of deflation and inflation at Pu'u 'Ō'ō was typically repeated at Kīlauea's summit, albeit with time delays of few hours to approximately half a day [*Wolfe et al.*, 1987], as shown in Figure 4. At Kīlauea's summit, changes in the volume of stored magma produced pressure changes within a shallow reservoir of 0.9±0.7 km^3 in volume [*Johnson*, 1992; *Ohminato et al.*, 1998; *Poland et al.*, 2009, 2012; *Gonnermann et al.*, 2012] and located beneath the southeastern edge of Halemau'ma'u crater [*Almendros et al.*, 2002; *Cervelli and Miklius*, 2003]. It is referred to as the Halemau'ma'u reservoir. These pressure decreases resulted in deflationary summit tilt changes and were presumably a consequence of increased magma flow rates into the ERZ, caused by the eruptive magma withdrawal at Pu'u 'Ō'ō. Subsequent refilling of the coupled Kīlauea-ERZ-Pu'u 'Ō'ō magmatic system during repose periods resulted in a gradual increase in pressure and in inflation.

3.2. Delay Times

[35] We hypothesize that the differences in time for the onset of deflation and reinflation between Pu'u 'Ō'ō and Kīlauea's summit were a consequence of the time required for changes in magma pressure to be transmitted through the elastically deforming dike, which hydraulically connected the shallow magma body beneath Pu'u 'Ō'ō with

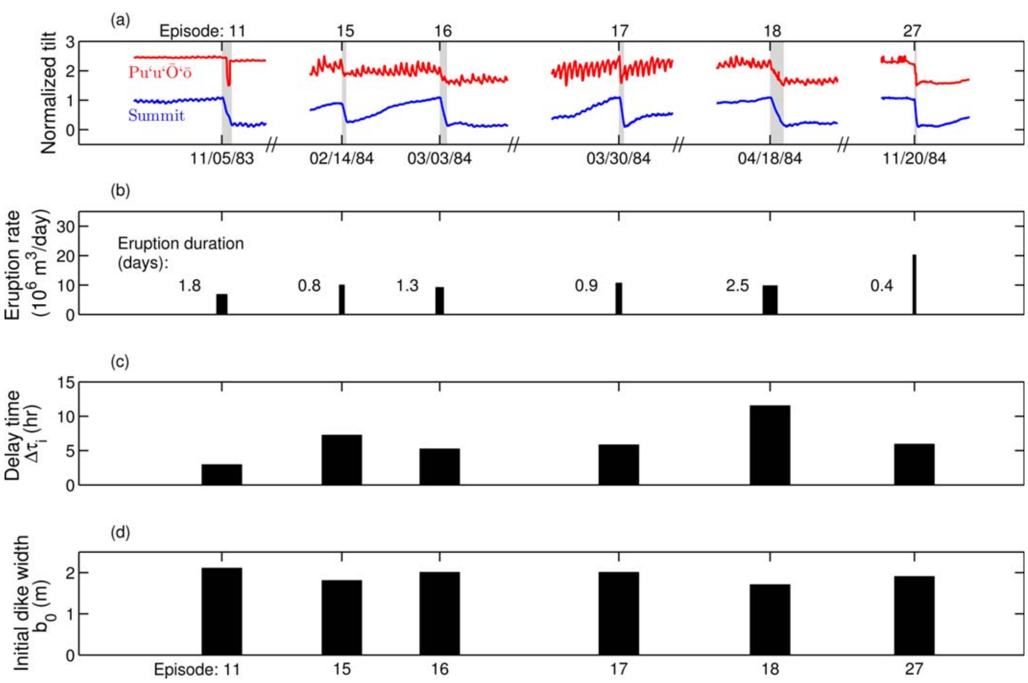

Figure 4. (a) Tilt data at Kīlauea's summit (blue) and at Pu'u 'Ō'ō (red) for selected episodes, normalized to maximum tilt amplitude. Gray bars indicate eruptive periods. (b) Eruption rate and duration. Bar heights represent erupted volumes and widths represent eruption durations, also indicated by numbers. (c) Reinflation delay time, $\Delta\tau_i$,

Kīlauea's summit [*Thurber*, 1984; *Dvorak and Okamura*, 1987; *Johnson*, 1992; *Barker et al.*, 2003; *Poland et al.*, 2013, Figure 3]. Figure 4 shows those episodes for which tilt data at both Pu'u 'Ō'ō and Kīlauea's summit have a distinct deflation-inflation pattern. Delay times for the onset of reinflation, $\Delta\tau_i$, were typically easier to quantify than deflation delay times, $\Delta\tau_d$, and were therefore chosen as the key observational constraint for our analysis.

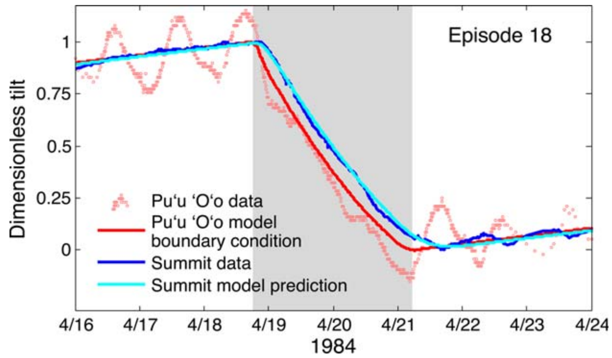

Figure 5. Observed and predicted normalized tilt at Pu'u 'Ō'ō and at Kīlauea's summit for episode 18 (shaded area indicates the eruptive period). Red filled circles are normalized tilt at Pu'u 'Ō'ō and the red line is the smoothed fit for the model boundary condition at Pu'u 'Ō'ō (equation (15)). The blue line is the normalized tilt at Kīlauea's summit,

3.3. Modeling Episode 18

[36] To illustrate the utility of the elastic dike model for characterizing the transport properties of the ERZ, we present detailed model results for episode 18 of the Pu'u 'Ō'ō-Kupaianaha eruption. Because the deflationary patterns are similar among all the episodes shown in Figure 4, we show in section 3.5 how the range in observed delay times from different episodes can be modeled by varying model parameters.

3.3.1. Boundary Conditions

[37] Because of the aforementioned uncertainties in geometry, we refrain from modeling the Pu'u 'Ō'ō magma storage reservoir explicitly. Instead, we use existing estimates of pressure variations during the early Pu'u 'Ō'ō eruptive episodes [*Hoffmann et al.*, 1990] as scaling factors to obtain a relationship between the observed tilt changes and the time-varying pressure changes at the Pu'u 'Ō'ō end of the dike. Consistent with conventional linearly elastic kinematic deformation models [*Mogi*, 1958], we assume that the total pressure variation during the eruptive episode, Δp, is proportional to the total tilt variation, $\Delta\psi$, and to the total change in stored volume of magma, ΔV. This proportionality relation is valid only if, as we assume, the magma within the system is incompressible. Changes in tilt, ψ, are caused by volume changes in the reservoir, so that $\partial_t\psi \propto Q_{\mathrm{Puu}} - Q_{\mathrm{e}}$, where Q_{Puu} is the time-dependent flow out of the dike and Q_e is the eruptive flow rate. We use the dimensionless tilt, defined as $\hat{\psi}(t) = \psi(t)/\Delta\psi$ [*Dvorak and Okamura*, 1985], to obtain the change in pressure with respect to time at the Pu'u 'Ō'ō end of the dike

$$\partial_t p = \Delta p \partial_t \hat{\psi}. \tag{14}$$

[38] This normalization of ψ avoids some of the difficulties associated with obtaining reliable absolute values for ERZ tilt amplitudes for the time period between 1983 and 1985 (A. Miklius, *pers. comm.*).

[39] To model episode 18 we use a value of $\Delta p =$ 0.34 MPa, which represents the independently estimated average value of deflationary pressure change during the early Pu'u 'Ō'ō eruptive episodes [*Hoffmann et al.*, 1990]. It should be noted that the estimates of Δp do not correlate with other parameters, such as the length of the eruptive episode [*Hoffmann et al.*, 1990]. Furthermore, precise values of Δp do not significantly affect our results, because the delay time is relatively insensitive to the magnitude of pressure variations (section 3.5).

[40] The pressure boundary condition at the Pu'u 'Ō'ō end of the dike, that is $x = 0$, is thus given as

$$p(t) = p_0 + 0.34\mathrm{MPa} \cdot \hat{\psi}_{\mathrm{smooth}}(t), \tag{15}$$

where $\hat{\psi}_{\mathrm{smooth}}$ is a linear fit to the normalized tilt, $\hat{\psi}$, at $\tau_{\mathrm{i}} \leq t \leq \tau_d$. For the time period $\tau_d < t < \tau_{\mathrm{i}}$ we use a smooth cubic spline fit to the dimensionless tilt, constrained to match the linearly fitted tilt at $t = \tau_d$ and $t = \tau_i$ (Figure 5). At the onset of deflation at Pu'u 'Ō'ō, that is $t = \tau_d$, we have $\hat{\psi} = 1$ and at the onset of reinflation, that is $t = \tau_i$, the value is $\hat{\psi} = 0$. Different methods of processing the raw tilt data to remove noise and diurnal variations, such as sinusoidal corrections, do not provide any improvement over the fit employed here.

[41] At Kīlauea's summit, which corresponds to $x = 15$ km, we impose a Dirichlet boundary condition. The specified value of $p = p_H$ is based on the

Table 2. Values of Parameters for the Best Fit to Episode 18 Tilt Data

Parameter	Value
dike height a	3 km
magma viscosity μ	100 Pa s
friction coefficient γ	12
shear modulus of host rock G	3 GPa
Poisson's ratio for host rock ν	0.25
elasticity of the host rock λ	$7.5 \cdot 10^{-7}$ m/Pa
initial dike width b_0	1.6 m
amplitude of pressure variation Δp	0.34 MPa
delay time for the onset of deflation $\Delta\tau_d$	3.5 h
delay time for the onset of reinflation $\Delta\tau_i$	11.5 h
eruption duration T_E	2.5 days

pressure within the Halemaʻumaʻu reservoir, p_H, which is calculated from its change in volume due to magma flow into the dike. The relationship between this change in volume and the change in pressure is based on the assumption that deformation is the consequence of volume and pressure changes of incompressible magma within an uniformly pressurized spherical reservoir embedded in a homogeneous rock medium [*Mogi*, 1958]. The resultant relationship between pressure and volume change is

$$\partial_t p_H = \frac{4G}{3V_H}\partial_t V_H, \tag{16}$$

where $V_H = 0.9$ km^3 is the assumed volume of the Halemaʻumaʻu reservoir (section 3.1).

3.4. Model Results

[42] The model is calibrated to match the observed tilt at Kīlauea's summit by adjusting b_0, a, and G. These variations correspond to specific changes in the transport properties of the ERZ, as discussed later. A best fit is obtained by grid search, minimizing the difference between predicted and observed onset of reinflation at Kīlauea's summit. The observed and modeled normalized tilts for episode 18 are shown in Figure 5. In addition to matching the overall shape of the observed tilt curve, the model reproduces both delay times $\Delta\tau_d = 3.5$ h and $\Delta\tau_i = 11.5$ h. Table 2 lists the model parameters used to obtain the best fit to the data for episode 18. The predicted time-delayed pressure change at Kīlauea's summit is of similar magnitude as that imposed at Puʻu ʻŌʻō (Figure 6a) and would result in a few tens of μrad deflationary tilt change at Uwēkahuna tilt station, consistent with the magnitude of the east-west component of tilt variations recorded during the eruptive episode [*Wolfe et al.*, 1987; *Parfitt and Wilson*, 1994].

[43] Figure 6a shows predicted pressure variations within the dike for episode 18 as a function of time and position between Puʻu ʻŌʻō and Kīlauea's summit, whereas Figure 6b shows the corresponding predicted magma flow rate $q = abu$. Magma withdrawal during the eruptive episode causes a decrease in magma pressure at Puʻu ʻŌʻō. The resultant increase in pressure gradient, $\partial_x p$, causes an increase in magma flow rate, q. Mass balance requires that both the pressure change and "surge" in magma flow propagate along the ERZ from Puʻu ʻŌʻō to Kīlauea's summit. The time required to reach the summit results in a delay in the onset of deflation at the summit, $\Delta\tau_d$, relative to Puʻu ʻŌʻō. Given the estimated size of the Halemaʻumaʻu reservoir, mass balance calculations indicate that during the eruptive episode, only a modest volume of magma, of the order of 10^5 m^3, was withdrawn from the Halemaʻumaʻu reservoir. The majority of the erupted volume was derived from within the deflating dike, combined with magma stored within the Puʻu ʻŌʻō reservoir.

[44] Once there is no more magma withdrawal at Puʻu ʻŌʻō, tilt changes reverse from rapid deflationary to slow inflationary, because of gradual magma flow from Kīlauea's summit into the dike and shallow Puʻu ʻŌʻō reservoir. This is represented in the model by the gradual increase in specified pressure at the Puʻu ʻŌʻō end of the dike. The change in $\partial_t p$ from deflation to inflation travels uprift from Puʻu ʻŌʻō to Kīlauea's summit and arrives with a slightly longer delay, $\Delta\tau_i$, than the

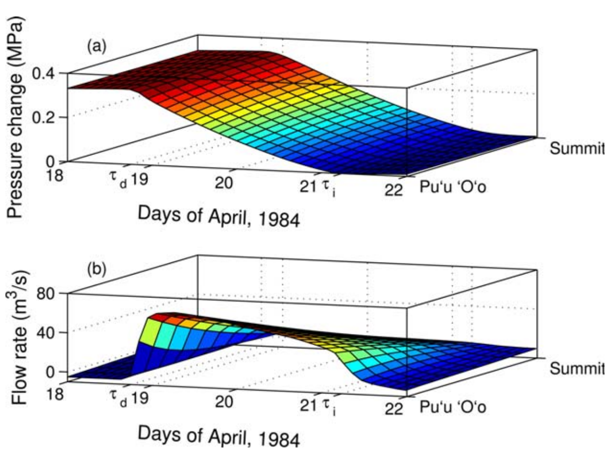

Figure 6. (a) Modeled change in magma pressure within the dike during episode 18 as a function of horizontal position

deflation delay. This increase in delay time is a consequence of the elastic decrease in dike width (a function of magma pressure), which results in more viscous resistance to magma flow and, hence, a decrease in the pressure diffusivity of equation (6).

3.5. Sensitivity to Model Parameters

[45] Although we show the modeled pressure and tilt changes for episode 18 only, the model was applied to other episodes as well. To illustrate the changes in model parameters that are required to match the range in observed delay times, we provide a sensitivity analysis of model predictions to parameters. This analysis not only shows the trade-offs between parameters, but it also demonstrates how relatively subtle variations thereof can result in changes in delay times that span the whole range of observed values.

[46] Relevant parameters are Δp (magnitude of deflation), T_E (duration of the eruption), λ (elasticity of the dike), and b_0 (initial dike width), whereas μ (magma viscosity) is relatively well constrained and not expected to change significantly. b_0 is of key importance, because it provides a quantitative measure of the pressure diffusivity, κ_0, and the characteristic transport efficiency, ϕ_0, of the ERZ magma pathway during the early eruptive episodes. Changes in delay times are most strongly affected by changes in ϕ_0 (Figure 7), which we speculate to be caused by changes in b_0, perhaps due to thermal erosion of dike walls or to freezing of magma against them during repose periods [*Costa et al.*, 2007]. Because it is unlikely that the dike-like ERZ magma pathway was of uniform geometry, as is our idealized model, the predicted change in ϕ_0 should be viewed as representative of either an average change throughout the ERZ or of a spatially localized change, perhaps due to opening or removal of a constriction or wall-rock collapse [*Parfitt and Wilson*, 1994].

[47] Figure 7 also demonstrates that changes or uncertainty in either T_E or Δp will not significantly affect predicted delay times, but that model results are sensitive to λ. We used $\lambda = 7.5 \cdot 10^{-7}$ m Pa^{-1}. Given an assumed dike height of 3 km [e.g., *Wolfe et al.*, 1987; *Koyanagi et al.*, 1988; *Okamura et al.*, 1988; *Parfitt and Wilson*, 1994; *Heliker et al.*, 2003] and $\nu = 1/4$, this would correspond to a shear modulus of 3 GPa. Although the effective shear modulus of Kīlauea's summit or ERZ are difficult to constrain [*Johnson*, 1987], Kīlauea is predominantly built of lava flows with

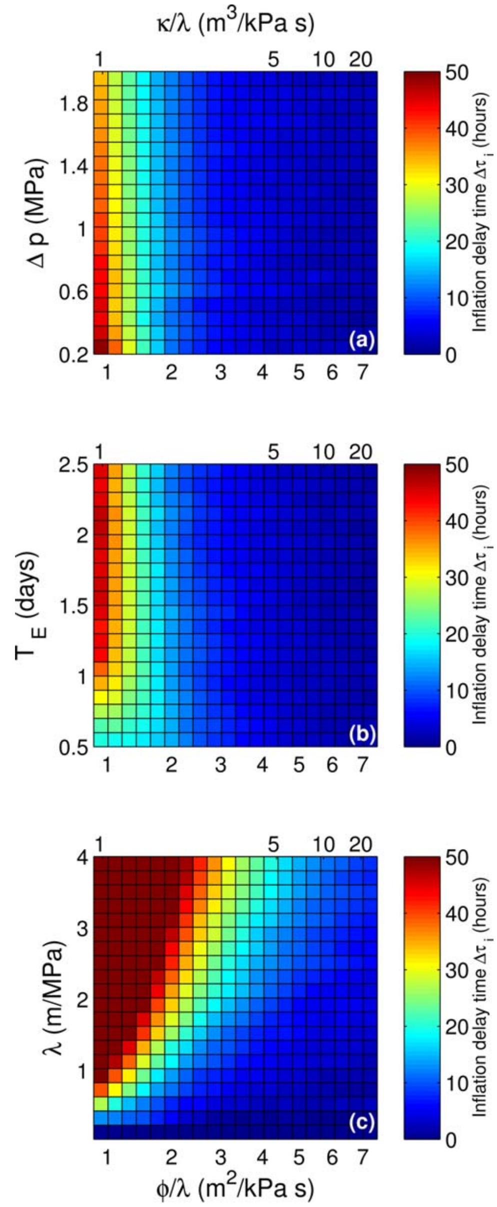

Figure 7. Parameter sensitivity of reinflation delay times, $\Delta\tau_i$. (a) $\Delta\tau_i$ as a function of dike width, b_0, and magnitude of pressure change, Δp. (b) $\Delta\tau_i$ as a function of dike width, b_0, and eruption duration, T_E. (c) $\Delta\tau_i$ as a function of dike width, b_0, and rock elasticity λ. The observed values of $\Delta\tau_i$ range between 1 and 12 h. All parameters not shown are kept constant at the values used for episode 18: $b_0 = 1.5$ m, $\Delta p =$ 0.34 MPa, $T_E = 2.5$ days, $a = 3$ km, $G = 3$ GPa, and $\mu =$

abundant void space [*Ryan et al.*, 1983], and its effective shear modulus is expected to be lower than the intrinsic shear moduli of laboratory-scale basalt samples [*Johnson*, 1987], which range between a few and several tens of gigapascals, depending on porosity [*Manghnani and Woollard*, 1968]. Because it is unlikely that the effective value of λ will change significantly between individual episodes, the observed changes in delay time are best explained by changes in b_0.

[48] Observed delay times during the early Puʻu ʻŌʻō eruptive episodes range between 3 and 12 h (Figure 4). Figure 7 indicates that these delay times can be modeled for dike widths between 1.7 and 2.1m, a variation of less than 20%. Delay time also increases somewhat with the duration of the eruption (Figure 7). Although there is no unequivocal correlation between episode duration and delay time for the episodes shown in Figure 4, it should be noted that episode 18 has the longest delay time as well as the longest duration.

[49] The elastic dike model does not account for magma compressibility, which is expected to increase the compliance of the system [*Rivalta and Segall*, 2008]. Using an incompressible approximation, delay times become very short as λ decreases (Figure 7). This is a consequence of the large increase in stiffness of the combined magma-wall rock system. For gas-poor magmas, the dominant effect on the overall compliance of the system is the elasticity of the wall rock (Appendix A).

[50] The volume fraction of exsolved magmatic gases within the ERZ is poorly constrained. Although the majority of exsolved magmatic volatiles, predominantly CO_2, is thought to escape to the surface before entering the ERZ [e.g., *Poland et al*., 2012, and references therein], some volatile bubbles may be transported into the ERZ. Of these, only bubbles smaller than $\sim$1–10 mm in diameter will rise buoyantly at significantly smaller velocities than the average horizontal flow velocity of magma within the dike and, thus, get transported for significant distances within the ERZ. Changes in magma pressure within the dike are unlikely to result in sufficient volatile exsolution to significantly affect magma compressibility (Appendix A). Therefore, although the ERZ magma may contain some fraction of exsolved volatiles, it is unclear if there was a sufficient volume of exsolved volatiles to significantly affect our overall model results. However, explicit inclusion of magmatic volatiles and magma compressibility are desirable model enhancements for future work along these lines.

4. Conclusions

[51] We have shown that a one-dimensional model of magma flow in an elastic-walled dike can reproduce spatiotemporal patterns of tilt at Kīlauea Volcano during the early eruptive episodes of the ongoing Puʻu ʻŌʻō-Kupaianaha eruption. Simplifying assumptions of the model are an idealized uniform dike geometry and an incompressible magma (Appendix A). Furthermore, equation (4) is strictly valid only for long-wavelength pressure pertubations and far from the dike tip [*Dunham and Ogden*, 2012]. Although future work along these lines should eliminate some of these approximations, our results, despite potential inaccuracies, demonstrate that spatiotemporal deformation patterns at active volcanoes bear important information about magma transport properties of the magmatic plumbing system. In particular, the time delays of deflation and inflation between the Puʻu ʻŌʻō vent and Kīlauea's summit can be explained by the propagation of pressure transients within such an elastically deformable dike.

[52] The delay times depend on the dike dimensions, the elasticity of the wall rock, the magma viscosity, as well as the amplitude and duration of the deflationary phases themselves. The most important of these parameters is the dike width, which is a measure of the characteristic magma transport efficiency. Consequently, delay times and changes thereof can be used to provide constraints on the average magma transport properties of the ERZ.

[53] Since the eruption started in 1983, the ERZ has seen other dike intrusions [*Segall et al*., 2001; *Montgomery-Brown et al*., 2010] as well as changes in the eruptive style [*Heliker and Mattox*, 2003]. However, the pathway connecting the summit to the Puʻu ʻŌʻō vent has remained an effective means for magma transport. Similar to the delay in deflation described herein, Kīlauea's summit is now experiencing episodes of deflation and inflation on the time scales of hours to days [*Cervelli and Miklius*, 2003], which are also being recorded along the ERZ, albeit with time delays relative to the summit. Although the dike that is thought to have existed during the early eruptive episodes may have evolved into a more pipe-like conduit [*Cervelli and Miklius*, 2003], the propagation of pressure changes in an elastically deforming conduit may also be a viable explanation for these more recent observations.

[54] Similar records of delayed transmission of pressure variations have been measured elsewhere, for example, during the 1960 Kapoho eruption at Kīlauea [*Eaton*, 1962], as well as during the 1984 eruption of Krafla Volcano in Iceland

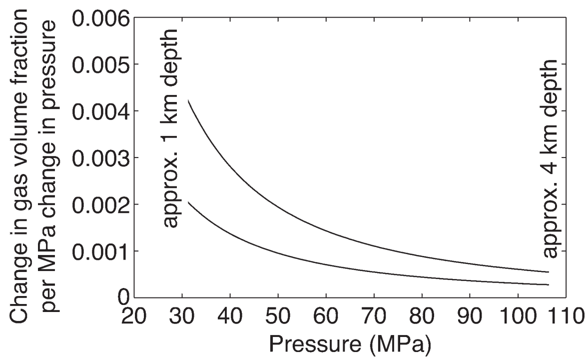

Figure A1. Fractional change in volume of exsolved volatiles per 1 MPa change in pressure over a range of pressures corresponding to the approximate depth range of the ERZ dike that is approximately 1–4 km depth. The upper curve is for water-saturated conditions, assumed to be 7000 ppm, whereas the lower curve is for water concentrations in equilibrium with a CO_2-rich volatile phase with a CO_2 mole fraction of 0.99.

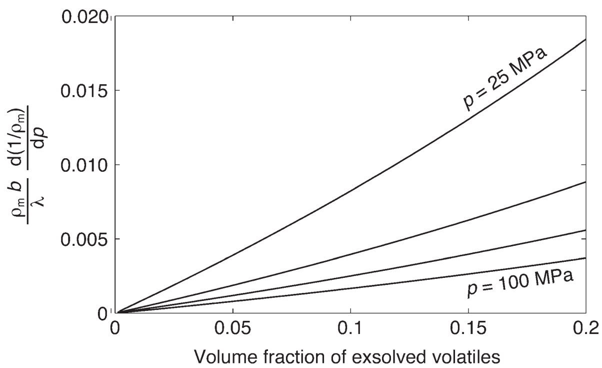

Figure A2. $\Lambda = \omega/(2\pi G^* b\beta_m)$, the ratio of the magma compressibility, β_m, to the elastic compressibility of the dike [*Dunham and Ogden*, 2012], scaled by the wavelength of the pressure pertubation ω, for different values of the volume fraction of exsolved volatiles, where $G^* = G/(1-\nu)$. Curves are for pressures of 25, 50, 75, and 100 MPa, roughly spanning the range in depths from the top to the bottom of the ERZ dike. Calculations are based on a CO_2-rich gas phase and the equation of state of *Holloway* [1977] with modifications by *Flowers* [1979]. For wavelengths $\omega > 1000$ m, the value of $\Lambda \gg 1$, indicating that magma compressibility is negligible.

[*Tryggvason*, 1986]. The model presented here could also provide constraints on the transport properties of the magmatic pathways during those eruptions.

Appendix A: Compressibility

[55] Magma supply to Kīlauea's summit is thought to constitute an open system wherein bubbles of exsolved magmatic volatiles decouple from the melt phase and rise buoyantly to the surface [e.g., *Poland et al.*, 2012, and references therein]. Consequently, it is not clear what fraction of the already exsolved volatiles will actually enter the ERZ [e.g., *Gerlach*, 1986]. Magma is thought to flow into the ERZ at a few kilometers depth [*Cervelli and Miklius*, 2003; *Montgomery-Brown et al.*, 2010; *Poland et al.*, 2013] where, assuming that the melt is equilibrated with CO_2 rich bubbles derived from deeper within the system, the dissolved CO_2 content of the magma will be a few hundred parts per million, and the H_2O and S content will be in the thousands parts per million. Most of this H_2O and S will be lost upon eruption at the Pu'u 'Ō'ō vent and is thought to play a dominant role in the upper few hundred meters of eruptive ascent during sustained fountaining episodes [*Wilson et al.*, 1980; *Head and Wilson*, 1987, 1989]. Here we provide a brief analysis of the effect of volatile exsolution due to expected pressure changes associated with the early eruptive episodes at Pu'u 'Ō'ō as well as of the role of magma compressibility due to the presence of exsolved volatiles.

[56] Figure A1 shows the expected fractional change in volume of exsolved volatiles at a given pressure for a decrease in pressure of 1 MPa. The calculations account for the combined exsolution of CO_2, H_2O, and S and are based on the solubility models of *Gerlach* [1986] and *Dixon* [1997], together with ideal gas behavior. Local changes in pressure, as modeled within the dike, are <1 MPa and will result in approximately a 0.1% change in the volume of exsolved volatiles.

[57] *Dunham and Ogden* [2012] define the compressibility ratio as the ratio of the compressibility of the elastic dike, β_d, to the compressibility of the magma, β_m as

$$\Lambda = \frac{\beta_d}{\beta_m} = \frac{\omega}{2\pi G^* b\beta_m}. \tag{A1}$$

[58] Here $G^* = G/(1-\nu)$ and ω is the wavelength of the pressure pertubation. For values of $\Lambda \gg 1$, magma compressibility is negligible. Figure A2 indicates that this is the case for pertubation wavelengths greater than 1000 m. As shown in Figure A2, the pressure pertubation extends along the entire length of the dike (15 km), hence, the approximation of an incompressible magma is not unreasonable.

[59] In the long-wavelength approximation, *Woods et al.* [2006] provide a formulation for compressible magma flow within an elastic-walled dike, which indicates that magma compressibility cannot be neglected if $d(1/\rho_m)/\mathrm{d}p \sim \lambda/(\rho_m b)$. Figure A3 shows the ratio of $d(1/\rho_m)/\mathrm{d}p$ to $\lambda/(\rho_m b)$ as a function of the volume fraction of exsolved volatiles, assuming ideal gas behavior and λ equal to the value used for the Pu'u 'Ō'ō model (Table 2). It can be seen that for a wide

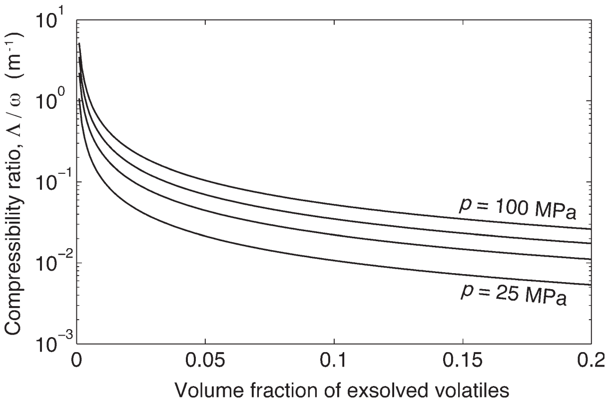

Figure A3. Ratio of $d(1/\rho_m)/\mathrm{d}p$ to $\lambda/(\rho_m b)$ as a function of the volume fraction of exsolved volatiles. Curves are for pressures of 25, 50, 75, and 100 MPa, roughly spanning the range in depths from the top to the bottom of the ERZ dike. Calculations are based on a CO_2-rich gas phase and the equation of state of *Holloway* [1977] with modifications by *Flowers* [1979]. For a wide range of exsolved gas content $d(1/\rho_m)/\mathrm{d}p \ll \lambda/(\rho_m b)$ and based on the formulation of *Woods et al*. [2006], the effect of magma compressibility should be negligible.

range of conditions the effect of magma compressibility is small, because of the compliance of the large-aspect ratio dike.

[60] Based on Figures A1–A3, it can be concluded that our assumption of negligible magma compressibility is first order justified.

Appendix B: Viscous Friction Term

[61] The friction term that appears in the momentum equation (equation (2)) can be derived from the unidirectional steady flow between two parallel plates that extend indefinitely along the x and z directions. The momentum equations for flow in the x direction are [*White*, 1991]

$$0 = -\partial_x p + \mu \partial_y^2 v_x, \tag{B1}$$

$$0 = -\partial_y p. \tag{B2}$$

[62] Here v_x is the velocity along the x direction, varying along the y direction, perpendicular to the plates, whereas $v_y = v_z = 0$. From equation (B2) we find that $p = p(x)$, so that equation (B1) can be integrated to give

$$v_x = \frac{1}{2}\mu \partial_x p \left(y^2 + Ay + C\right). \tag{B3}$$

[63] The boundaries are at $y = 0$ and $y = b$, where $v_x = 0$, which gives $A = -b$ and $C = 0$. Thus, the velocity has a parabolic profile given by

$$v_x = \frac{1}{2}\mu \partial_x p \left(y^2 - by\right). \tag{B4}$$

[64] The mean velocity is $u = b^2/(12\mu\partial_x p)$, and the pressure drop due to viscous friction is

$$\partial_x p = -\frac{12\mu u}{b^2}. \tag{B5}$$

Appendix C: Inertial Forces

[65] The scaling used to nondimensionalize equation (6) can be used to evaluate the relative importance of the individual terms appearing in the momentum conservation equation (equation (2)). The assumption of a viscously dominated flow is justified if viscous forces are much larger than inertial forces. The first term of the right-hand side of equation (2) is the fluid pressure gradient

$$A = \partial_x p \sim \frac{1}{\lambda}\frac{B_0}{L_0}. \tag{C1}$$

[66] The second right-hand side term in equation (2) is the viscous force, which can be written as

$$B = \frac{\gamma \mu u}{b^2} \sim \frac{1}{\lambda}\frac{B_0}{L_0} \tag{C2}$$

for a characteristic flow velocity of $U_0 = L_0/T_0$ [*Bokhove et al*., 2005].

[67] Both A and B are of the same order. The left-hand side of equation (2), denoted as C, comprises fluid acceleration and inertial forces and is given by

$$C = \rho_m(\partial_t u + u\partial_x u) \sim \rho_m \beta^2 B_0^3 \left(\frac{B_0}{L_0}\right)^3. \tag{C3}$$

[68] The ratio between inertial and dissipative forces is

$$\frac{C}{A} \sim \frac{C}{B} \sim \rho_m \beta^2 \lambda \frac{B_0^5}{L_0^2}. \tag{C4}$$

[69] Reasonable values of parameters that are applicable to Kīlauea are $\rho_m \sim 2500$ kg/m^3, $\lambda \sim 10^{-6}$ m^2 s^2 kg^{-1}, $\beta \sim 10^3$ m^{-1} s^{-1}, $L_0 \sim 15$ km, and $B_0 \sim 1$ m. The resultant force balance results in a ratio $C/A \sim C/B \sim 10^{-5}$, which justifies our assumption of neglecting the inertial terms in equation (2).

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